MAXWELL DISTRIBUTION OF GAS MOLECULE VELOCITIES ©

Computational Document

by

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Adapted from a document by Flick Coleman, Wellesley College MA
The original document is included in the set of documents on the Mathcad Web page.

$R := 8.314$

$M := 0.028$

Some constants: R in SI units, molecular weight for N_2 in kg per mole.

$umax := 2000$

Maximum speed for calculation

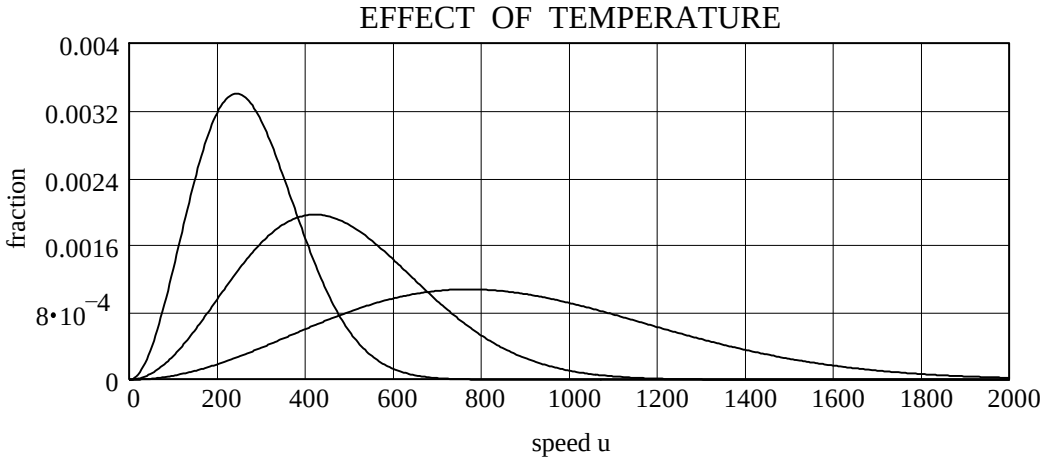
$i := 0 .. 500$ $\quad u_i := \frac{i \cdot umax}{500}$

Calculate for 500 points

$T := \begin{pmatrix} 100 \\ 300 \\ 1000 \end{pmatrix}$ $\quad j := 0 .. 2$

An array of temperatures

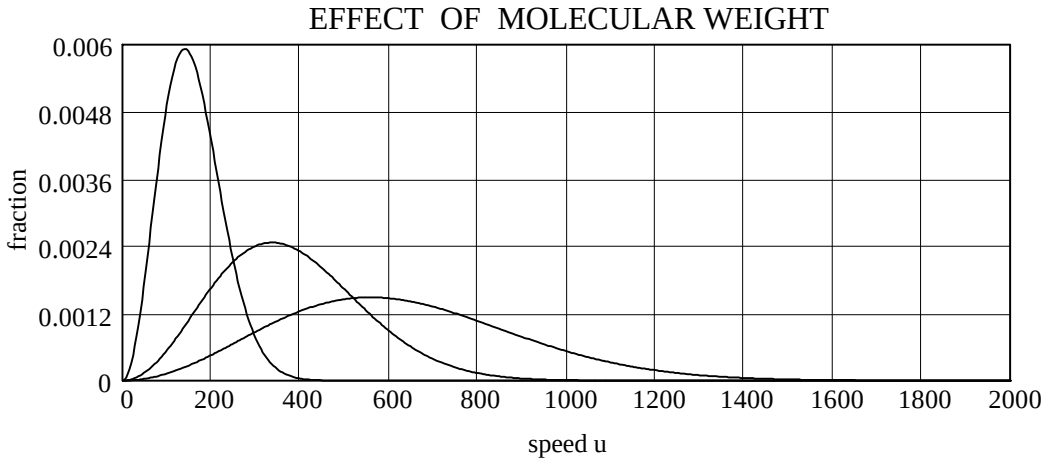
$$P_{i,j} := 4\cdot\pi\cdot\left(\frac{M}{2\cdot\pi\cdot R\cdot T_j}\right)^{\frac{3}{2}}\cdot\exp\left[\frac{-M\cdot\left(u_i\right)^2}{2\cdot R\cdot T_j}\right]\cdot\left(u_i\right)^2$$ Maxwell's function



Now we test the effect of molecular weight on the speeds of CH_4, CO_2, and I_2 all at a temperature of 300°K.

$$M := \begin{pmatrix} 0.016 \\ 0.044 \\ 0.254 \end{pmatrix} \qquad j := 0..2 \qquad T := 300$$

$$P_{i,j} := 4\cdot\pi\cdot\left(\frac{M_j}{2\cdot\pi\cdot R\cdot T}\right)^{\frac{3}{2}}\cdot\exp\left[\frac{-M_j\cdot(u_i)^2}{2\cdot R\cdot T}\right]\cdot(u_i)^2$$



Next we integrate the Maxwell function

$m := 0.016$

$T := 320$

Select the molecular weight and temperature.
Pick a very large velocity v=50000 for upper limit.
We had to switch from u to v and M to m to keep MATHCAD happy.

$$Int := \int_{0}^{50000} 4\cdot\pi\cdot\left(\frac{m}{2\cdot\pi\cdot R\cdot T}\right)^{\frac{3}{2}}\cdot\exp\left(\frac{-m\cdot v^2}{2\cdot R\cdot T}\right)\cdot v^2\, dv$$

$Int = 1$

Use the expression below for using calculus to find the maximum of the Maxwell distribution. Follow instructions from your sheets.

$$4\cdot\pi\cdot\left(\frac{m}{2\cdot\pi\cdot R\cdot T}\right)^{\frac{3}{2}}\cdot\exp\left(\frac{-m\cdot u^2}{2\cdot R\cdot T}\right)\cdot u^2$$